



# Mathematics, Science & Technology

## PART II.9

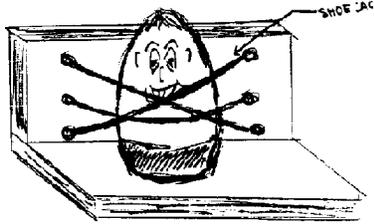
Bill and Ted's Eggsellent Adventure.....2

**NOTE:** This document is a work in progress. Parts II and III, in particular, are in need of further development, and we invite the submission of additional learning experiences and local performance tasks for these sections. Inquiries regarding submission of materials should be directed to: The Mathematics, Science, and Technology Resource Guide, Room 681 EBA, New York State Education Department, Albany, NY 12234 (tel. 518-474-5922).



<http://www.nysed.gov>

Standards & Performance Indicators



# Bill and Ted's Eggcellent Adventure

**MST**

**1**

- ▲ investigation/technological invention
- ▲ creative solutions
- ▲ work schedules/plans
- ▲ perform test

**MST**

**2**

- ▲ prepare multimedia
- ▲ model solutions

**MST**

**5**

- ▲ thorough investigation
- ▲ creative solutions
- ▲ work schedules/plans
- ▲ devise test solutions
- ▲ use equipment correctly
- ▲ CADD
- ▲ selecting systems

**MST**

**6**

- ▲ mathematical models
- ▲ predictions

**MST**

**7**

- ▲ design solutions
- ▲ work effectively
- ▲ gather/process information
- ▲ generate/analyze ideas
- ▲ observe common themes
- ▲ realize results
- ▲ present results

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Grades 11&12

*Engineers have to go through many designs in order to optimize their solution. To design a crumple zone isn't as easy as it seems. It took me maybe five tries just to get a design decent enough to optimize. By the time I had finished making my crumple zone, it wasn't fancy anymore, but it was very efficient.*

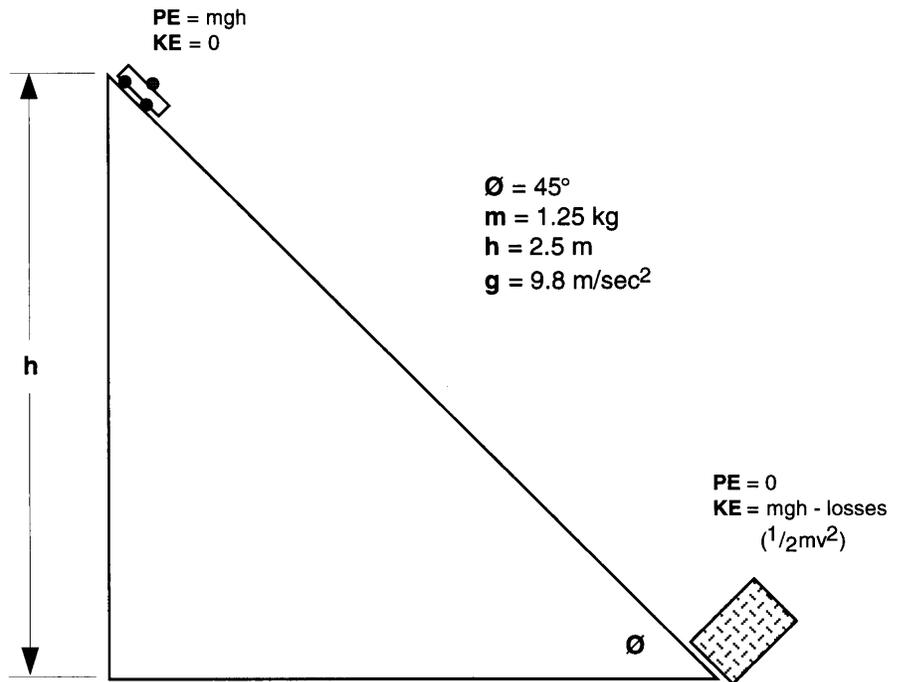
Student



## Introduction to the Design Activity

In this problem solving and design activity you will use elements of physics, mathematics, and technology to design safety systems that will protect the *occupants* of a vehicle. The vehicle will be a standard dynamics cart (purchased from Frey Scientific). The occupants will be two eggs (Bill and Ted). The vehicles will be tested by sending them down a ramp and allowing them to slam into a concrete block at the bottom.

Each team of students will design the following systems: a restraint system, a crumple zone system, an ergonomics system, and a car body system. The design solutions will be modeled and tested. This activity will utilize the *energy model* to determine whether or not the safety systems designed (restraint and crumple zone) will allow the occupants to survive a crash test.



When each vehicle is placed on the top of the ramp, it will have a certain amount of stored, or **potential energy** ( $PE$ ) that is a function of the mass of the vehicle ( $m$ ), the height of the ramp ( $h$ ) and the acceleration due to gravity ( $g$ ). This can be expressed as:

$$PE = mgh$$

$PE$  is the potential energy in joules;

$m$  is the mass in kilograms;

$h$  is the height in meters;

$g$  is the acceleration due to gravity (a constant 9.8 meters/second<sup>2</sup>).

**Energy** is defined as the ability to do work, or the ability to put an object into motion. **Work** is defined as the amount of force exerted on an object to move it a specified distance. Work and energy are both measured in foot-pounds in US units, and in newton-meters in SI (International) units. The *joule* is an SI unit that is equivalent to one newton-meter. *Joules* will be used in all calculations of work and energy in this activity.

The  $PE$  of the vehicle can also be referred to as its GPE, or gravitational potential energy. The vehicle will have a certain number of joules of  $PE$  at the top of the ramp. The energy in motion, or **kinetic energy** ( $KE$ ) at the top of the ramp will be zero, until the vehicle is released. The car will accelerate as it travels down the ramp. The increase in the velocity of the car will relate to an increased  $KE$ , and a decreased  $PE$ . At the bottom of the ramp, the velocity of the vehicle will be maximum, as will its kinetic energy. The potential energy at the bottom will be zero, indicating that all of the  $PE$  has been converted into  $KE$ .

The kinetic energy can be expressed as:

$$KE = 1/2 mv^2$$

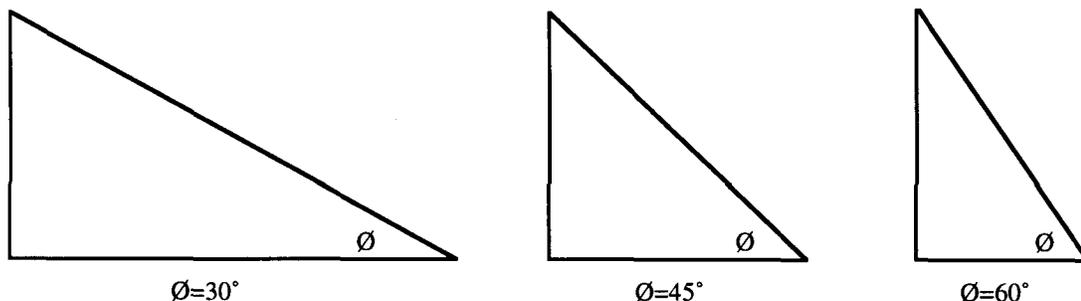
*KE* is the kinetic energy in joules;

*m* is the mass in kilograms;

*v* is the velocity on meters per second.

This is of course theoretical; all of the potential energy will be not converted into kinetic energy of the vehicle ( $1/2 mv^2$ ) at the bottom of the ramp. Energy will be lost due to friction (between the road surface and the wheels, and between the axles and wheels). Rotational losses will also be a factor (the energy used to turn the wheels) in the final velocity at the bottom of the ramp. These frictional and rotational losses can be significant, and must therefore be considered.

Another important consideration is the angle of the ramp. Although the acceleration due to gravity is a constant ( $9.8 \text{ m/s}^2$ ), the angle of the ramp will control the actual acceleration of the vehicle. The vehicle acceleration will be higher as the ramp is made steeper. We can relate this to the "thrill" of accelerating down a steep section on a roller coaster. The illustration below shows three ramps at different angles; the height is held constant for each ramp. The ramp at  $30^\circ$  will accelerate the vehicle the least, and the  $60^\circ$  ramp will accelerate the vehicle the most.



If the same vehicle was placed at the top of all three ramps, it would have the same amount of potential energy ( $mgh$ ) which would be converted into kinetic energy as it travels down the ramp (including losses). The actual distance traveled along the ramp will differ for each angle; as the ramp steepens, the distance the vehicle travels becomes less. Using the relationships for *work* and *force*, it can be shown that when the vehicle travels a shorter distance (on the  $60^\circ$  ramp), the force of the vehicle upon impact will be higher than it would be on the  $30^\circ$  or  $45^\circ$  ramp. This is evidenced by the greater magnitude of impacts that occur at steeper angles. The acceleration of the vehicle varies proportionally as the angle of the ramp is changed.

The actual acceleration of the vehicle is proportional to the **sine** of the angle of the ramp. The acceleration of the vehicle down the ramp equals the product of the acceleration due to gravity and the sine of the angle:

$$a = g \sin\theta$$

The  $g \sin\theta$  values for each of the three angles above is:

$$30^\circ \text{ ramp: } 9.8 (.500) = 4.90 \text{ m/s}^2$$

$$45^\circ \text{ ramp: } 9.8 (.707) = 6.93 \text{ m/s}^2$$

$$60^\circ \text{ ramp: } 9.8 (.866) = 8.49 \text{ m/s}^2$$

The above example illustrates the significant differences in acceleration that occur as the angle of the ramp is varied. These differences in acceleration will translate into proportionally different forces at impact.

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## Inquiry, Analysis and Design

Since the energy model is used in this case study, an analysis of the forces exerted on either the vehicle or the occupants is unnecessary. Resolving the forces (using vectors) requires a much more involved analysis. Since the acceleration due to gravity is a constant ( $9.8 \text{ m/s}^2$ ), tests will be performed by determining the amount of PE ( $mgh$ ) each safety system can absorb.

Asuggested apparatus to test the restraint and crumple zone safety systems is described below. The apparatus consists of guides fabricated from 1" PVC pipe, sleeves fabricated from 1 1/4" PVC pipe, a wooden base (5/4x6) and platen (2x6), and a variety of pipe flanges, elbows, and couplings. The PVC guides are 1-meter long, and spaced approximately 0.5 meters apart. The specimen is tested by allowing the platen to drop a specified distance. With the weight of the platen known, the distance that the platen falls (including the compressed portion of the specimen) can be used to calculate the potential energy ( $mgh$ ) absorbed by the specimen. Weights can be added to the platen to increase the  $mgh$  without increasing the height of the apparatus. Tests on each of the safety systems can be analyzed, and the results used to optimize the design solution.

Eleven members of each design team are expected to actively participate in all of the facets of this design and problem solving activity. Although all responsibilities will be shared, each team member will be placed in charge of a specific subsystem. The *Chief Engineer* for each subsystem will be held accountable for the operation and final success of that subsystem.

### Subsystem Chief Engineers:

- **Restraint Subsystem Chief Engineer:** Responsible for "occupants" being held in a safe/secure position during and after the collision.
- **Crumple Zone Chief Engineer:** Responsible for the modeling of the subsystem to insure that the design is adequate for the predictable forces and energy transfers at impact.
- **Car Body Design Chief Engineer:** Responsible for the design and construction of a realistic looking car body.
- **Ergonomics Chief Engineer:** Responsible for the *human factors* considerations which include: entry/exit, visibility, and space considerations.

Design teams will determine the forces and energy transfers for their vehicle. They will then determine the resulting energy that will be absorbed by their safety systems.

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## Assessment of Work by Design Teams

- Portfolio documenting and describing the design: Research, investigations, report, drawings
- Optimization of design solutions
- *Ergonomics* of the design
- Realistic component of the car body design
- Presentation/justification of design to class
- Survival of the occupants (*Bill and Ted*)
- Student Journal/Log Book
- Class participation.

*Each Design Team must submit a portfolio which documents their work on this case study. The portfolio must contain each of the following materials:*



## Restraint Subsystem

- Sketches of all preliminary designs
- Materials chosen and rejected (analysis)
- Test results (mgh) for each trial
- Reasons for failure: observations, causes/effects
- Methods of optimization and results
- Final optimized solution: drawings/description.

## Crumple Zone Subsystem

- Sketches of all preliminary designs
- Materials chosen and rejected (analysis)
- Test results (mgh,  $mg\Delta h$ ) for each trial as a function of height and/or angle
- Reasons for failure: observations, causes/effects
- Methods of optimization and results
- Final optimized solution: drawings/description.

## Car Body Design Subsystem

- Sketches of all preliminary designs
- Materials chosen and rejected (analysis)
- Design goals stated: appearance, aerodynamics, ergonomics, weight, and crash-worthiness.

## Ergonomics Subsystem

- Measurements and investigations on human factors engineering
- Methods used to apply ergonomics to final design
- Evidence of ergonomics in car body design.

## Restraint Subsystem Book

*Crumple Zone*

*Restraint Subsystem*

*Car Body*

*Ergonomics*

**Chief Engineer, Andy**

**Chief Engineer, Pat**

**Chief Engineer, Eric**

**Chief Engineer, Eric**



## Crumple Zone Design Group#5

Throughout the process of designing an efficient crumple zone, there was a key factor that was always kept in mind. Good absorption was the goal for the crumple zone.

The desired crumple zone had to absorb enough kinetic energy from reaching the rest of the car so that the remaining energy could be easily dissipated with the restraint systems. Many materials were tested, and their "behavior" was recorded. From these preliminary testings, the actual shape and forms of the crumple zone were formed.

Cardboard had a tendency to spring back no matter what shape it was made into. It was made into cylinders, tubes, cubes, pyramids, "I" bar shapes or even combinations tended to bounce, a lot.

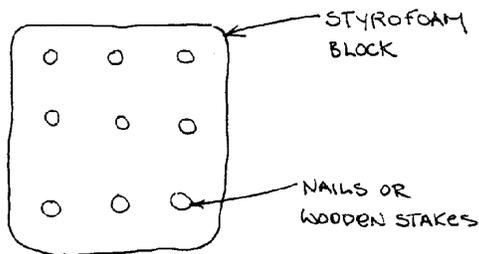
Paper a distant cousin of cardboard has less tendency to bounce, but still springs. The most efficient absorbing shape is the cylinder, or unsealed box. The sealed air tight box bounced the highest.

Plastic tendency bounce is comparable to paper in that it springs, but less than cardboard. All shapes sprung back up. Tubes, cylinders, and frustum shaped cut outs. The truncated cone shape worked the best.

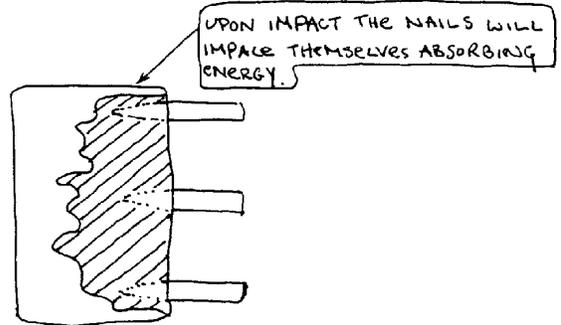
Metal which was the best material for its "unbounceability". No bounce was seen with any shape for the foil or can. The soda can did bounce without slits which had let air out. It actually. The soda can did bounce without slits which had let air out. It actually took effort for a bounce to appear such as using paper with metal. A pure metal crumple zone would probably not bounce. (The final design was of pure metal materials, soda can and foil.) Yet it can be noted that a pure metal crumple zone may not be very efficient in an actual collision with only the 3 inch thick dimensions.

From the sketches on the following pages, the solutions are ordered in preliminary, immediate and preliminary solutions.

# NAILS AND STYROFOAM



DESIGN #1



USING 14 NAILS AND TESTING IT:

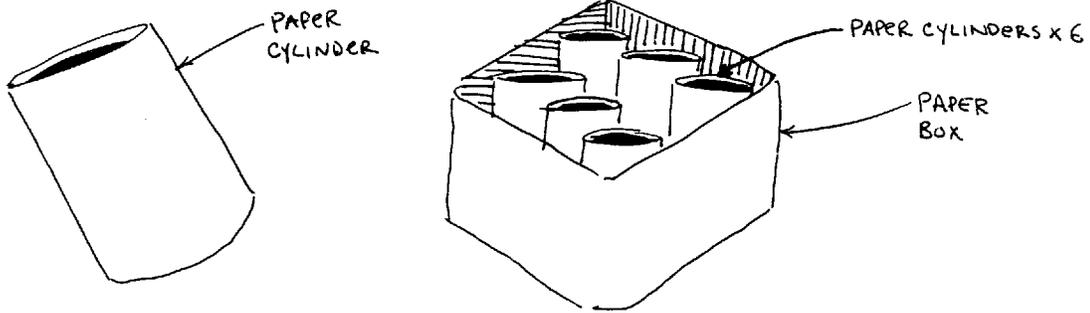
Tested twice  
with similar  
result.

$$J = MGH$$
$$J = (.745 \text{ m})(9.8 \text{ m/sec}^2)(2 \text{ Kg})$$
$$J = 14.602$$

- NO BOUNCE, BUT NAILS WENT COMPLETELY THROUGH TO THE ANVIL.

Design #2

# PAPER CYLINDERS



Test:

$$J = MGH$$

$$J = (2\text{kg}) (9.8) (.792)$$

m/sec<sup>2</sup>    M

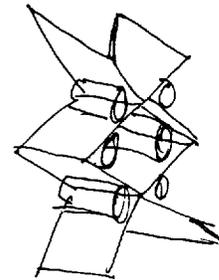
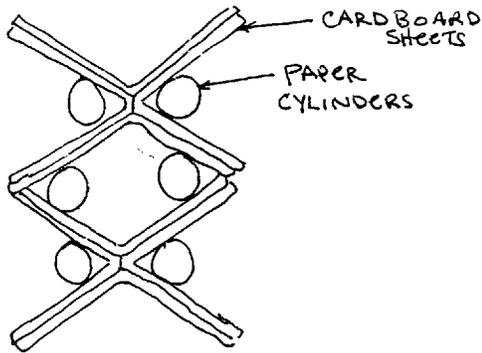
$$J = 15.5$$

BETTER RESULTS  
THAN NAILS AND STYROFOAM.

- SLIGHT BOUNCE NOTED  
THE BOX TRAPPED AIR.

No NAME

DESIGN #3



TEST:

$$Pe = MGH$$

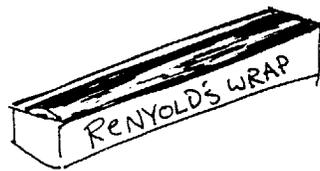
$$Pe = (9.8) (4.) (1.75) = 30.38$$

m/sec<sup>2</sup>    kg    m

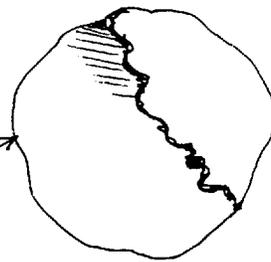
- GOOD ABSORPTION, BUT IT  
RANCHED.

# ALUMINUM BALL

Design # 4



BALL  
OF SHINY  
FOIL



8 cm

- Test:

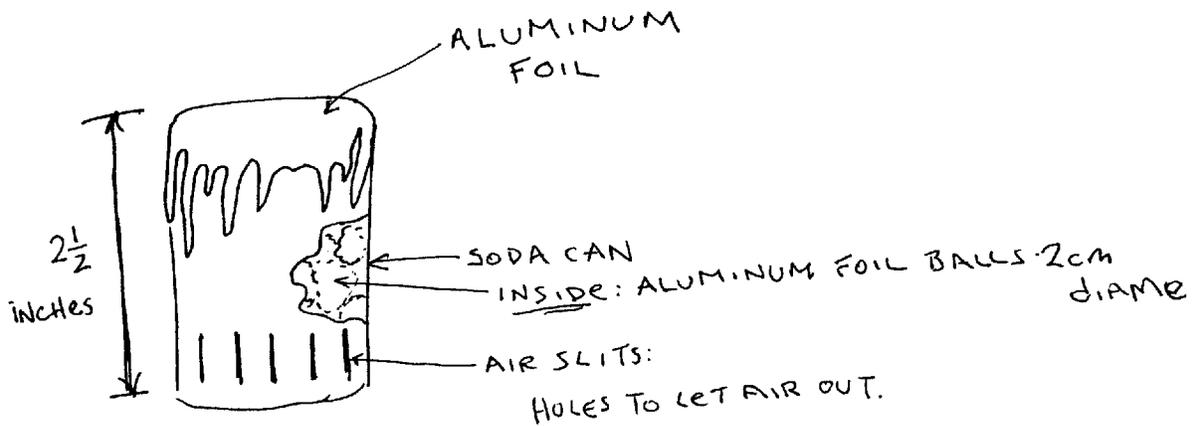
$$P_e = MGH$$

$$(74.5) \underset{\substack{\uparrow \\ M}}{(2 \text{ kg})} (9.8) \underset{\substack{\uparrow \\ \text{m/sec}^2}}{=} 14.6 \text{ J}$$

- No NOTICEABLE BOUNCE.

# KENETIC KILLER

FINAL



TEST:

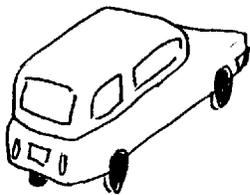
$$Pe = MGH$$

$$Pe = (1.65)(4\text{kg})(9.8)$$

m                      m/sec<sup>2</sup>

$$= 30.1 \text{ J}$$

- NO BOUNCE



## Restraint Subsystem

AS THE RESTRAINT SUBSYSTEM CHIEF ENGINEER, I WAS RESPONSIBLE FOR THE "OCCUPANTS" BEING HELD IN A SAFE / SECURE POSITION DURING AND AFTER THE COLLISION. OVER THE LAST COUPLE OF WEEKS WE HAVE TESTED OVER 5 DIFFERENT RESTRAINT SYSTEMS. USING ALL TYPES OF MATERIALS; EVERYTHING FROM RUBBER BANDS TO COTTON AND GAUZE.

WE FOUND THAT CERTAIN MATERIALS SUCH AS RUBBER BANDS AND PLASTIC WRAP DID NOT WORK PROPERLY. AND MATERIALS LIKE SHOE LACES AND GAUZE DID NOT WORK ON THEIR OWN, BUT WHEN WE INCORPORATED THEM TOGETHER IN ONE SUBSYSTEM, WE GOT A WORKING, EFFICIENT, FUNCTIONAL RESTRAINING DEVICE.

THE PROBLEM WITH THE SHOE LACES ALONE, WAS THAT THEY WERE TOO NARROW AND DID NOT HAVE ENOUGH SURFACE AREA IN CONTACT WITH THE OCCUPANTS. THEREFORE, WHEN A COLLISION OCCURED, THE SHOE LACES ACTED LIKE A KNIFE AND SLICED THROUGH THE OCCUPANTS. IN ORDER TO IMPROVE OUR DESIGN, WE INCREASED THE SURFACE AREA OF OUR RESTRAINING DEVICE BY GLUING THE SHOE LACES TO FOAM BOARD STRIPS OR CARD BOARD STRIPS. WE THEN WRAPPED THEM IN COTTON AND GAUZE TO CUSHION THE OCCUPANTS DURING THE COLLISION. WE ALSO ADDED PADDING TO THE BACK OF THE SEAT BY USING FOAM BOARD AND GAUZE, TO PROTECT THE OCCUPANTS WHEN THEY ARE "WHIP LASHED" BACK INTO THE SEAT AFTER THE INITIAL IMPACT. BY ADDING COTTON IN AND BEHIND THE EGG CANTON SEAT, WE ALSO LESSONED THE BLOW OF THE CRASH.

OUR ORIGINAL DESIGN HAS COME A LONG WAY TO THE FINISHED PRODUCT. IN THE BEGINNING OUR PRODUCT BORDERED ON OUR ERGONOMIC LIMITS, BUT IN THE FINAL PRODUCT, WE FOUND THAT WE DID NOT NEED ALL THE COTTON AND GAUZE. WE CUT BACK ON THE EXTRA PADDING AND WE HELPED OUR ERGONOMIC STATUS.

I THINK THAT WE ALL PUT A GOOD EFFORT INTO OUR DESIGN, AND AFTER TESTING IT WE KNOW IT WILL BE SUCCESSFUL. WE ALL ENJOYED WORKING ON OUR CAR, AND ARE PROUD OF OUR FINAL PRODUCT.

# Restraint Subsystem

## RESTRAINT SYSTEM!

TEAM #5

ANDY TAN  
ERIC LEE  
PAT MURPHY

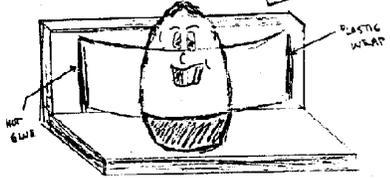
THEORY #1

BODY SHIELD THEORY!

$$PE = mgh$$

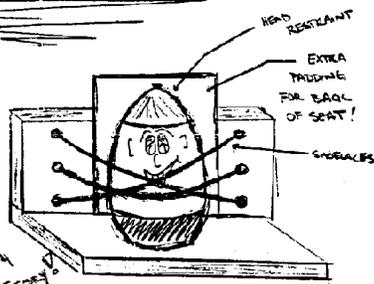
$$PE = (0.05 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (4 \text{ m})$$

$$PE = 1.96 \text{ J}$$



THEORY #3/#4

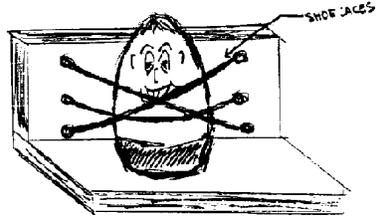
COMBO THEORY!



$$PE = mgh$$

$$PE = (0.05 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (1.6 \text{ m})$$

$$PE = 0.794 \text{ J}$$



THEORY #2  
SEATBELT THEORY!

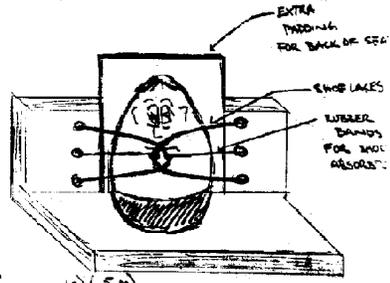
$$PE = mgh$$

$$PE = (0.05 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (4 \text{ m})$$

$$PE = 1.96 \text{ J}$$

\* FINAL \*  
SOLUTION  
(SEE COMPUTER PRESENT)

THEORY #5  
RUBBER BAND THEORY!



$$PE = mgh$$

$$PE = (0.05 \text{ kg}) \times (9.8 \text{ m/s}^2) \times (1.5 \text{ m})$$

$$PE = 0.735 \text{ J}$$

SALE & TECHNICAL SKILLS  
PRINCIPLES OF ENGINEERING  
MR. BOGALWE  
PERIOD 5

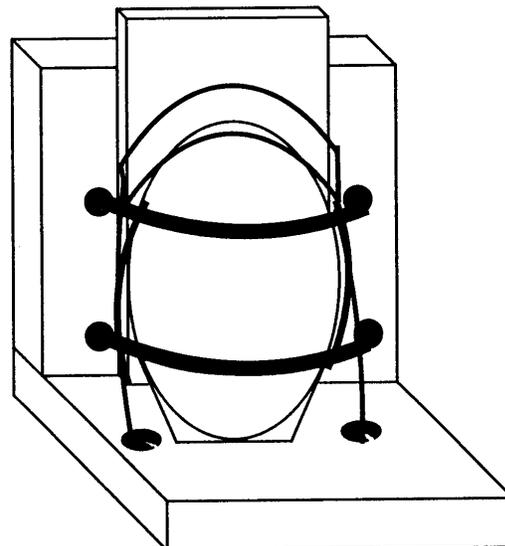
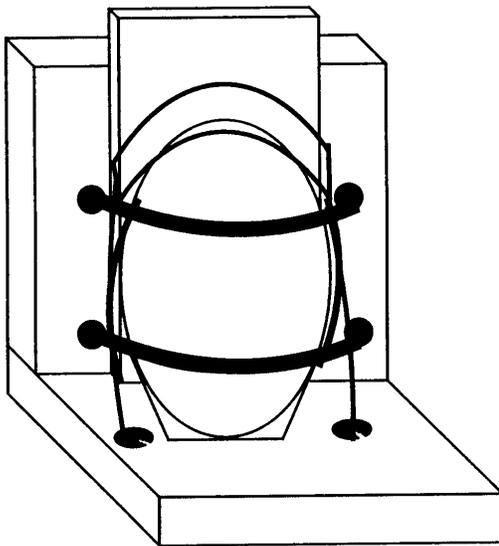
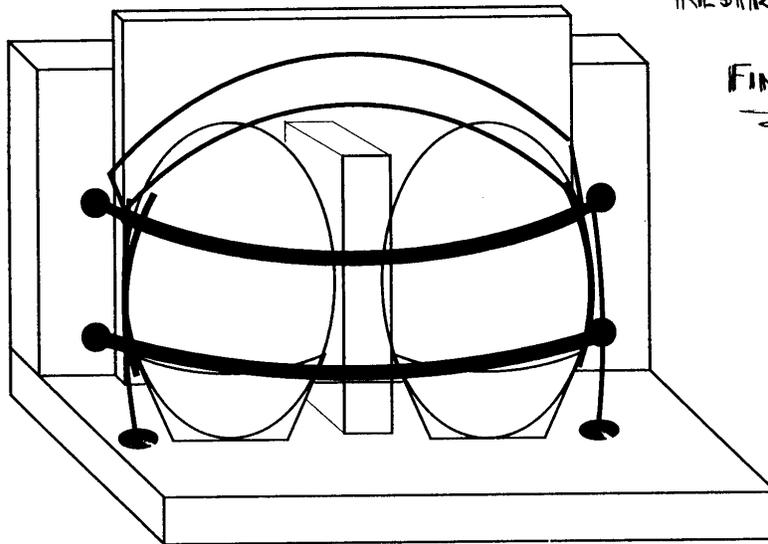
### RESTRAINT SYSTEM EVOLUTION

RESTRAINT SUBSYSTEM CHIEF ENGINEER  
PATRICK MURPHY

Restraint Subsystem

RESTRAINT SYSTEM

FINAL DESIGN! 



FINAL  
CALCULATIONS! 

$$P.E. = mgh$$

$$P.E_o = (.65 \text{ kg})(9.8 \text{ m/s}^2)(.9 \text{ m})$$

$$P.E. = .441 \text{ J (MAX.)! $$

## *Car Body*

Our car body designs were made up in order to have less gravity but weighs heavier than others, so that it will increase the speed of the car which will be dropped from the wooden rail. I chose to make the car heavier than anybody else in the class because I had a complete trust in Andy's crumple zone, Pat's restrain system, and wanted extra points added to our grades.

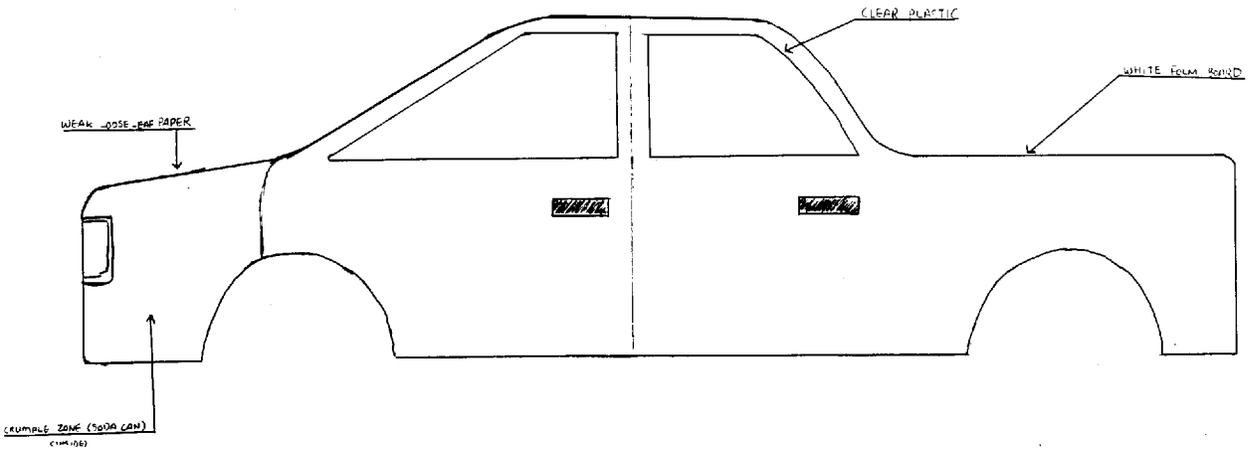
Our car body is made from a white form board which was painted black in order to bring out the style of the car. We agreed on the black color and went with it. I put clear plastics on the windows to stop the air from coming in and slowing down the car. Before I came up with the pick up truck idea I was going to make our design a mini van or a 4 by 4. We had to throw this ideas away because it was not suitable for our crumple zone and our restrain system. Besides our idea was an original, not like all the other groups who built a mini van.

As I mentioned at the presentation our main focus was at the crumple zone and at the restrain system. The car body had nothing to do with the protection of the eggs. This car body is only used for it's figure.

Bill and Ted will survive if they choose to ride in our car. I'm pretty sure that other groups will survive but I guarantee that our car will be most efficient.

# Car Body

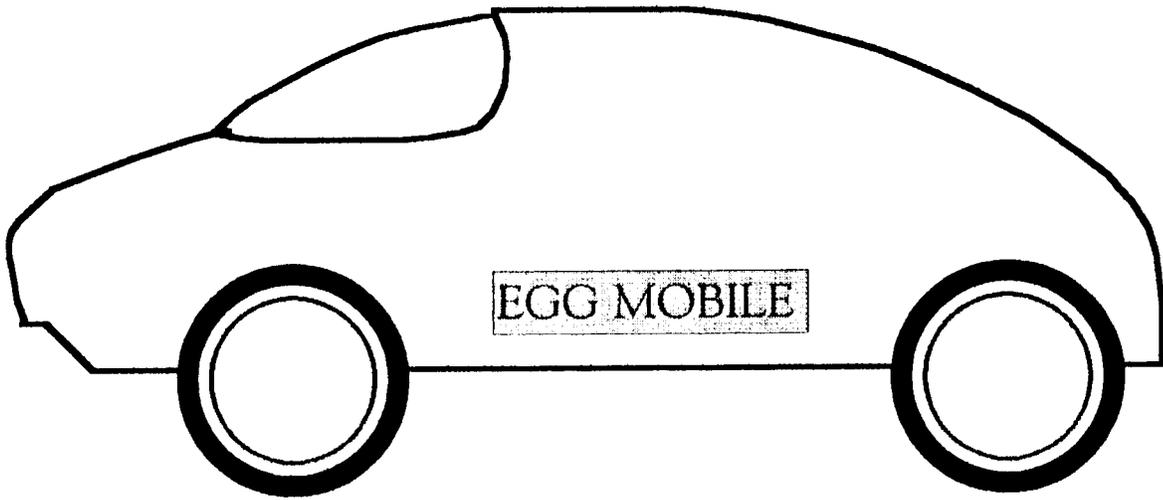
Final Carbody Drawing



## Car Body

### Primary Drawing #11

too round, it slows down. This led to an idea that we should make a sharper edged cars.



Auto Safety Case Study  
**Log Book Assessment**  
“Bill and Ted’s Eggsellent Adventure”

**Student:** \_\_\_\_\_

**Assessment Scale:**

- 4: Mastery:** Your work demonstrates excellence in this portion of the activity.
- 3: Accomplished:** Your work fulfills all of the objectives of this portion of the activity.
- 2: Acceptable:** Your work is acceptable, but needs minor revisions.
- 1: Unacceptable:** Your work is either incomplete, or requires major revisions.

- 1. Entries are made on a daily basis to document student’s work. \_\_\_\_\_
- 2. Entries are thorough and complete containing all relevant material. \_\_\_\_\_
- 3. Entries contain data, observations and an analysis of the investigations. \_\_\_\_\_
- 4. Entries indicate use of problem solving and design techniques. \_\_\_\_\_
- 5. Illustrations are included where appropriate, and they enhance the clarity of the logbook entries. \_\_\_\_\_
- 6. Entries demonstrate student’s contribution to the group effort in this activity. \_\_\_\_\_

**Total:** \_\_\_\_\_

**Auto Safety Case Study**  
**Assessment of Activity**  
“Bill and Ted’s Eggsellent Adventure”

*Design Team # \_\_\_\_ Subsystem Chief Engineers:*

Restraint Subsystem: \_\_\_\_\_ Crumple Zone Subsystem: \_\_\_\_\_

Car Body Subsystem: \_\_\_\_\_ Ergonomic Subsystem: \_\_\_\_\_

**Assessment Scale:**

- 4: Mastery:** Your work demonstrates excellence in this portion of the activity.
- 3: Accomplished:** Your work fulfills all of the objectives of this portion of the activity.
- 2: Acceptable:** Your work is acceptable, but needs minor revisions.
- 1: Unacceptable:** Your work is either incomplete, or requires major revisions.

**A. Restraint Subsystem**

- a) Preliminary designs are clearly identified with sketches. \_\_\_\_\_
- b) All data collected for preliminary designs, calculations (analysis of data), and observations are clearly documented for each trial. \_\_\_\_\_
- c) Design shows evidence of optimization. \_\_\_\_\_
- d) Final solution is provided with suitable drawings. \_\_\_\_\_
- e) Aconcise, written description documents your work. \_\_\_\_\_

**B. Crumple Zone Subsystem**

- a) Preliminary designs are clearly identified with sketches. \_\_\_\_\_
- b) All data collected for preliminary designs, calculations (analysis of data), and observations are clearly documented for each trial. \_\_\_\_\_
- c) Design shows evidence of optimization. \_\_\_\_\_
- d) Final solution is provided with suitable drawings. \_\_\_\_\_
- e) Aconcise, written description documents your work. \_\_\_\_\_

**C. Car Body Design Subsystem and Ergonomics Subsystem**

- a) Preliminary designs are clearly identified with sketches. \_\_\_\_\_
- b) Safety systems are incorporated into a realistic car-body design. \_\_\_\_\_
- c) Car body illustrates quality workmanship, and good utilization of both materials and equipment. \_\_\_\_\_
- d) Effective use of ergonomics in restraint system design. \_\_\_\_\_
- e) Visible indications that human factors engineering is incorporated into the vehicle design. \_\_\_\_\_

**D. Survival of the Occupants (Bill & Ted)**

- Bill and Ted survive the crash unharmed (no cracks): (4) \_\_\_\_\_
- Either Bill or Ted is injured (shell cracked) in the crash: (3) \_\_\_\_\_
- Bill and Ted are both injured in the crash: (2) \_\_\_\_\_
- Either Bill or Ted *eggspires* (cracks with leakage) in the crash: (1) \_\_\_\_\_

**E. Presentation of Design to Class**

- a) Presentation was organized and well-planned. \_\_\_\_\_
- b) Presentation was thorough and included all relevant content material. \_\_\_\_\_
- c) Responses to questions were clear and appropriate. \_\_\_\_\_
- d) Design was justified to class in presentation. \_\_\_\_\_
- e) All design team members actively participated in presentation. \_\_\_\_\_
- f) Presenters handled themselves in a professional manner. \_\_\_\_\_
- g) Presentation included a variety of audiovisual media and visual aids. \_\_\_\_\_

**F. Classwork/Groupwork**

- a) Student shows consistent effort. \_\_\_\_\_
- b) Work started in a businesslike manner at bell. \_\_\_\_\_
- c) Willingness to help other students. \_\_\_\_\_
- d) Suitable class conduct displayed. \_\_\_\_\_
- e) Student actively contributes to the group effort. \_\_\_\_\_
- f) Student actively participates in the peer review process. \_\_\_\_\_
- g) Student completes work in a timely fashion (at specified deadlines). \_\_\_\_\_
- h) Student worked responsibly as a subsystem Chief Engineer, and was supportive to the other members of the design team. \_\_\_\_\_

**Student's Name:** \_\_\_\_\_